



Introduction to

Health Physics

Fifth Edition

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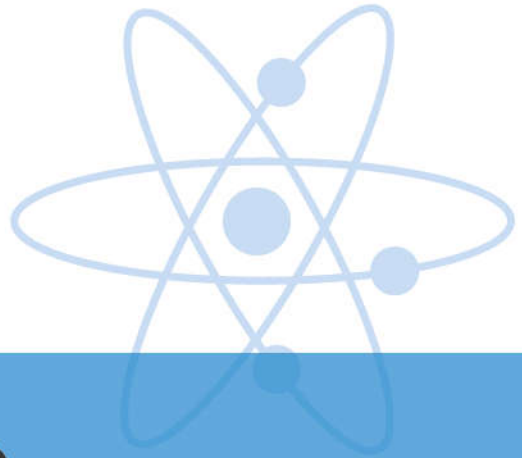
Thomas E. Johnson



Introduction to
Health Physics

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Introduction to Health Physics

Fifth Edition

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**Mc
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Education

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ISBN: 978-0-07-183526-8

MHID: 0-07-183526-1.

The material in this eBook also appears in the print version of this title: ISBN: 978-0-07-183527-5,

MHID: 0-07-183527-X.

eBook conversion by codeMantra

Version 1.0

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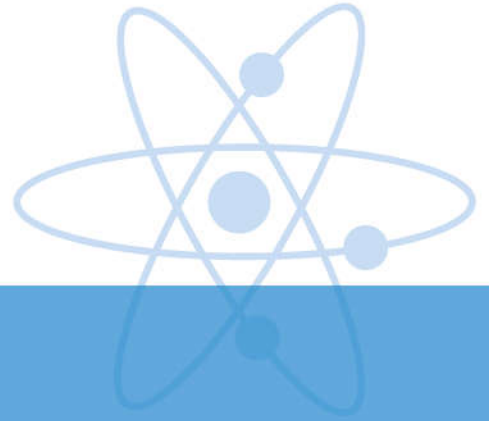
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*To Sylvia Cember
and to the memory of
Dr. Elda E. Anderson
and
Dr. Thomas Parran
To my wife, Melissa
and to the memory of
Dr. Herman Cember*

*“And whatever you do, in word or in deed, do everything in the name of the Lord Jesus, giving thanks
to God the Father through him.”*

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Preface

The practice of radiation safety is a continually evolving activity. Many of the changes in the practice of ionizing and nonionizing radiation safety, in calculation methodology, and in the methods for demonstrating compliance with the safety standards that have occurred since the publication of the previous edition of *Introduction to Health Physics* are incorporated in the fifth edition.

Since their inception in 1928, the Recommendations of the International Commission on Radiological Protection have formed the scientific basis for ionizing radiation safety standards issued by regulatory authorities throughout the world. Generally, earlier recommendations were successively more restrictive than the previous ones. The 2008 and 2015 recommendations, however, are essentially the same as the previous recommendations made in 1990. The main difference is that the 2008 and 2015 recommendations are made on the basis of the increased knowledge acquired since 1990 and for the first time, environmental protection is explicitly addressed. This is not surprising, since no harmful radiation effects have been observed among the population of radiation workers whose doses had been within the previous standards. The new recommendations continued to stress that all unnecessary exposure be avoided and that all exposures should be kept as low as reasonably achievable, economic, and social factors being taken into account. A reasonable question, therefore, that is raised by the ICRP recommendations is “How safe is safe?” This question lies in the field that Dr. Alvin Weinberg, the late director of the Oak Ridge National Laboratory, called *transscience*. Transscientific questions have a scientific basis, but they cannot be answered by science alone. Safety is a subjective concept that can be interpreted only within the context of its application. Policy decisions regarding matters of health and safety should be made in the context of public health. In the practice of public health, we find that numerous diseases and threats to health are always present in every community. The cost of controlling these threats to health is borne by the community. Since the community has limited resources, it must set priorities regarding which of the many real or perceived health threats to control. One of the techniques for quantifying the likelihood of the expression of a potential risk is called *quantitative risk assessment*. In the area of radiation safety, this usually deals with two main risks: (1) failure of a large technological system, such as a nuclear power plant, and (2) the long-term effects of low-level radiation. The results of quantitative risk assessment are often perceived as the determination of a real threat to life or limb, no matter how small the calculated chance of occurrence. However, quantitative risk assessment is

a calculation that almost always assumes the most pessimistic, and in many cases entirely unrealistic, values for parameters whose magnitudes include several different uncertainties. In addition to statistical uncertainties, for example, we must choose among several different equally reasonable models to which to apply the statistical data. One of the purposes of this edition is to provide the technical background needed to understand the calculation and use of quantitative risk assessment for radiation hazards in order to help us allocate our limited resources.

Although it has been a number of years since the ICRP recommended that health physics quantities be expressed in the meter–kilogram–second (MKS) units of the SI system rather than the traditional units based on the centimeter–gram–second (cgs) system, the change to the SI units has not yet been universally implemented in the United States. For example, the U.S. Nuclear Regulatory Commission continues to use the traditional system of units in its regulations. For this reason, this edition continues to use both systems, with one or the other equivalent quantity given in parentheses.

I also owe a debt of gratitude to Herman Cember, for allowing me to be a part of this book. I wish to thank Alex Brandl, Sanaz Hariri Tabrizi, Yuanlin Peng, and the many persons, too numerous to mention by name, for their helpful suggestions.

Thomas E. Johnson



Introduction

Health physics, radiation protection, radiological health, radiation safety, and radiological engineering are synonymous terms for the area of public health and environmental health engineering that deals with the safe use of ionizing and nonionizing radiation in order to prevent harmful effects of radiation to individuals, to population groups, and to the biosphere. The health physicist is responsible for safety aspects in the design of processes, equipment, and facilities utilizing radiation sources and for the safe disposal of radioactive waste so that radiation exposure to personnel is minimized and is at all times within acceptable limits; he or she must keep personnel and the environment under constant surveillance in order to ascertain that these designs are indeed effective. If control measures are found to be ineffective or if they break down, the health physicist must be able to evaluate the degree of hazard and make recommendations regarding remedial action.

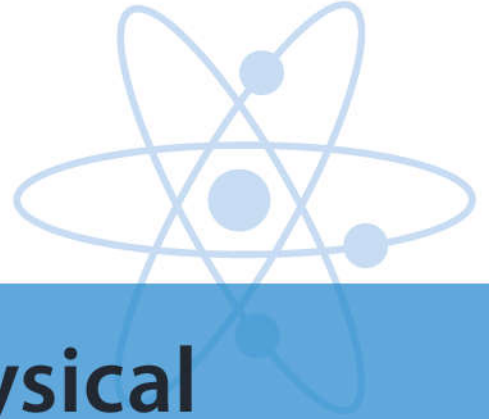
Public policy vis-à-vis radiation safety is based on political, economic, moral, and ethical considerations as well as on scientific and engineering principles. This textbook deals only with the scientific and engineering bases for the practice of health physics.

The scientific and engineering aspects of health physics are concerned mainly with (1) the physical measurements of different types of radiation and radioactive materials, (2) the establishment of quantitative relationships between radiation exposure and biological damage, (3) the movement of radioactivity through the environment, and (4) the design of radiologically safe equipment, processes, and environments. Clearly, health physics is a professional field that cuts across the basic physical, life, and earth sciences as well as such applied areas as toxicology, industrial hygiene, medicine, public health, and engineering. The professional health physicist, therefore, in order to perform effectively, must have an appreciation of the complex interrelationships between humans and the physical, chemical, biological, and even social components of the environment. He or she must be competent in the wide spectrum of disciplines that bridge the fields between industrial operations and technology on one hand and health science, including epidemiology, on the other. In addition to these general prerequisites, the health physicist must be technically competent in the subject matter unique to health physics.

2 CHAPTER 1

The main purpose of this book is to lay the groundwork for attaining technical competency in health physics. Radiation safety standards undergo continuing change as new knowledge is gained and as the public's perception of radiation's benefits and risks evolve. Radiation safety nomenclature too changes in order to accommodate changing standards.

Because of the nature of the subject matter and the topics covered, however, it is hoped that the book will be a useful source of information to workers in environmental health as well as to those who will use radiation as a tool. For the latter group, it is also hoped that this book will impart an appreciation for radiation safety as well as an understanding of the philosophy of environmental health.



Review of Physical Principles

MECHANICS

Units and Dimensions

Health physics is a science and hence is a systematic organization of knowledge about the interaction between radiation and organic and inorganic matter. The Health Physics Society defines *health physics* as “The science concerned with the recognition, evaluation, and control of health hazards to permit the safe use and application of ionizing radiation.” Quite clearly, the organization of health physics must be quantitative as well as qualitative since the control of radiation hazards implies knowledge of the dose–response relationship between radiation exposure and the biological effects of radiation.

Quantitative relationships are based on measurements, which, in reality, are comparisons of the attribute under investigation to a standard. A measurement includes two components: a number and a unit. In measuring the height of a person the result is given, for example, as 70 inches (in.) if the Imperial system of units is used or as 177.8 centimeters (cm) if the metric system is used. The units *inches* in the first case and *centimeters* in the second case tell us what the criterion for comparison is, and the number tells us how many of these units are included in the quantity being measured. Although 70 in. means exactly the same thing as 177.8 cm, it is clear that without an understanding of the units the information contained in the number above would be meaningless. In the United States, the imperial system of units (now U.S. customary units) is used chiefly in engineering, while the metric system is widely used in science.

The International Vocabulary of Metrology defines a quantity as “property of a phenomenon, body or substance where the property has a magnitude that can be expressed as a number and a reference.” The magnitude of the physical quantity is a numerical value, and the reference or standard will be the measurement unit, measurement procedure, or reference material. In the imperial system of units, these quantities are measured in feet, slugs (a slug is that quantity of mass that is accelerated at a rate of one foot per second per second by a force of one pound; a mass of 1 slug weighs 32.2 pounds), and seconds, respectively,

while the metric system is divided into two subsystems: the mks, in which the three quantities are specified in meters, kilograms, and seconds, and the cgs, in which centimeters, grams, and seconds are used to designate length, mass, and time.

By international agreement, the metric system has been replaced by the *Système International*, the International System of Units, or simply the SI system. Although many familiar metric units are employed in SI, it should be emphasized that SI is a new system and must not be thought of as a new form of the metric system. The International System of Quantities (ISQ, ISO/IEC 80000) has a set of seven base quantities from which all other quantities in the SI system are derived. All the other units such as force, energy, power, and so on are derived from the basic units of length (meter), mass (kilogram), time (second), electric current (ampere), thermodynamic temperature (Kelvin or Celsius), amount of substance (mole), and luminous intensity (candela). Measurement units are real scalar quantities that can be expressed in base units or derived units. Every measurement has an uncertainty associated with it, although it may or may not be given in all situations. Chapter 9 will discuss uncertainty, which for simplicity will be omitted in most calculations.

The basic unit for length, the meter, is also used for determining parameters such as displacement. Displacement is the change in position from one point, x_1 , to another, x_2 :

$$\Delta x = x_2 - x_1. \quad (2.1)$$

Displacement (Δx) over a time interval (Δt) expresses the average velocity (\bar{v}_{avg}) in units of meters per second:

$$\frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \bar{v}_{\text{avg}}.$$

where velocity is a vector with magnitude (speed) and direction. Speed is simply the total distance covered in a time interval, and the instantaneous speed (or velocity) is given by

$$\bar{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{d\bar{x}}{dt}.$$

The rate of change of velocity with time is acceleration, and utilizes units of meters per second per second ($\text{m}\cdot\text{s}^{-2}$):

$$\bar{a}_{\text{avg}} = \frac{\Delta \bar{v}}{\Delta t}; \quad \text{and} \quad a = \frac{d\bar{v}}{dt}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2}.$$

Deceleration will produce negative values for acceleration.

The derived unit of force, the newton (N), is defined as follows:

One *newton* is the unbalanced force that will accelerate a mass of one kilogram at a rate of one meter per second per second.

Expressed mathematically, the result is Newton's Second Law:

$$\text{Force} = \text{mass} \times \text{acceleration},$$

that is,

$$\vec{F}_{\text{net}} = m\vec{a}, \quad (2.2)$$

where the net force on a body is the vector sum of all forces on that body.

The units associated with force in newtons are

$$\vec{F} = \text{kg} \cdot \frac{\text{m/s}}{\text{s}}.$$

Since dimensions may be treated algebraically in the same way as numbers, the dimension for acceleration is written as m/s^2 . The units for force in units of newton (N), therefore, are

$$\text{N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}.$$

The unit of force in the cgs system is called the *dyne* ($1 \text{ dyne} = 10^{-5} \text{ N}$). A dyne is defined as the force required to accelerate one gram at a rate of one centimeter per second squared. Although all health physics measurements are readily expressed in SI units, the U.S. Nuclear Regulatory Commission continues to use the traditional cgs units in its regulatory activities and cgs units will occasionally be utilized to illustrate various concepts throughout the text.

Work and Energy

Energy is defined as the *ability to do work*. Since all work requires the expenditure of energy, the two terms are expressed in the same units and consequently have the same dimensions. Work W is done, or energy expended, when a force \vec{F} is exerted through some distance x :

$$W = \int \vec{F} \cdot d\vec{x}. \quad (2.3)$$

In the SI system, the *joule* (J) (named after the British scientist who measured the mechanical equivalent of heat energy) is the unit of work and energy and is defined as follows:

One *joule* of work is done when a force of one Newton is exerted through a distance of one meter. Since work is defined as the product of a force and a distance, the units for work and energy are as follows:

$$\text{Joule} = \text{newton} \cdot \text{meter} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}.$$

The unit of work or energy in the cgs system is called the *erg* and is defined as follows:

One *erg* of work is done when a force of one dyne is exerted through a distance of one centimeter. The joule is a much greater amount of energy than an erg.

$$1 \text{ joule} = 10^7 \text{ ergs}.$$

Although the erg is very much smaller than a joule, it nevertheless is very much greater than the energies encountered in the submicroscopic world of the atom. When working on the atomic scale, a more practical unit called the *electron volt* (eV) is used.

The *electron volt* is a unit of energy and is defined as follows:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ erg.}$$

When work is done on a body, the energy expended in doing the work is added to the energy of the body. The energy expended when work (W) is done to accelerate a body appears as kinetic energy in the moving body.

$$W = \int \vec{F} \cdot d\vec{x} = \int \vec{F} \cdot \vec{v} \cdot dt = \int \vec{v} \cdot d\vec{p} = \int \vec{v} \cdot d(m \cdot \vec{v}) \quad (2.4)$$

Assuming that the mass, m , is constant,

$$W = m \int \vec{v} \cdot d(\vec{v}) = \frac{1}{2} m \int d(\vec{v} \cdot \vec{v}) = \frac{1}{2} m \int dv^2 = \frac{1}{2} mv^2. \quad (2.5)$$

Kinetic energy (E_k) is defined as the energy possessed by a moving body as a result of its motion, or the work done accelerating an object.

$$W = \Delta E_k = E_{k,f} - E_{k,i} \quad (2.6)$$

When the body is initially at rest, $E_{k,i} = 0$, and bodies of constant mass m , moving “slowly” with a velocity v less than about 3×10^7 m/s, the kinetic energy, E_k , is given by

$$E_k = E_{k,f} = W = \frac{1}{2} mv^2. \quad (2.7)$$

Potential energy is defined as energy that a body possesses by virtue of its position in a force field. In the case where work was done in lifting a body, the mass possesses more potential energy at the higher elevation than it did before it was lifted. Work was done, in this case, against the force of gravity and the total increase in potential energy of the mass is equal to its weight, which is the force with which the mass is attracted to the Earth, multiplied by the height through which the mass was raised.

For example, if a mass is lifted from one elevation to another, the energy that was expended during the performance of the work is converted to potential energy (E_p):

$$\Delta E_p = -W = \int \vec{F} \cdot d\vec{x}. \quad (2.8)$$

The total energy of the body is equal to the sum of its potential energy and its kinetic energy:

$$E_t = E_p + E_k, \quad (2.9)$$

assuming no external forces act on the system, and there are no internal forces (such as friction). The total energy, E , of a system can only change by the amount of energy transferred to or from the system.

$$W = \Delta E_p + \Delta E_k + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} \quad (2.10)$$

For an isolated system,

$$\Delta E = \Delta E_p + \Delta E_k + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} = 0. \quad (2.11)$$

When the speed of a moving body increases beyond about 3×10^7 m/s, we observe interesting changes in their behavior—changes that were explained by Albert Einstein.

RELATIVISTIC EFFECTS REVIEW

According to the system of classical mechanics that was developed by Newton and the other great thinkers of the Renaissance period, mass is an immutable property of matter; it can be changed in size, shape, or state but it can neither be created nor be destroyed. Although this law of conservation of mass seems to be true for the world that we can perceive with our senses, it is in fact only a special case for conditions of large masses and slow speeds. In the submicroscopic world of the atom, where masses are measured on the order of 10^{-27} kg, where distances are measured on the order of 10^{-10} m, and where velocities are measured in terms of the velocity of light, classical mechanics is not applicable. Albert Einstein's special theory of relativity provides us with an explanation for these circumstances.

There are three main postulates of Einstein's special theory of relativity:

1. The velocity of light in a vacuum is constant at 299,792,458 m/s (for practical purposes, a value of 3.00×10^8 is used) relative to every observer in any reference frame.
2. He also postulated that the speed of light is an upper limit of speed that a material body can asymptotically approach, but never can attain. Photons travel at the speed of light in a vacuum and in all inertial reference frames, as they have no mass.
3. The physics are the same for observers within the all inertial reference frames. Classical mechanics, optics, and other physics basics remain the same for all observers within any inertial reference frame. Making measurements from different frames of reference requires us to consider relativity.

Although the laws of physics are the same for all observers in inertial reference frames, measured values may not be the same for all observers. For example, time intervals as measured in two reference frames will depend on both space and time separation. For example, the observer of a person moving at a velocity close to the speed of light will measure a time "dilation." The time interval, t' , observed in the moving frame (from the point of view of the stationary observer) will be longer than the time interval observed by that observer in his/her resting frame (t). The time dilation, t' , can be calculated using the following equations:

$$\Delta t' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Delta t. \quad (2.12a)$$

When the following are substituted:

$$\beta^2 = \frac{v^2}{c^2}, \quad (2.13)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}, \quad (2.14)$$

we obtain the following equation:

$$\Delta t' = \gamma \Delta t. \quad (2.12b)$$



EXAMPLE 2-1(a)

If the mean life (t) of a muon is 2.2×10^{-6} seconds, with a velocity of 99.88% of the speed of light, what would the mean life (t') appear to be from a person in the rest frame?

Solution

Substituting values into Eqs. (2.14) and (2.12b),

$$\Delta t' = \gamma \Delta t,$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - (0.9988)^2}} = 20.42,$$

$$\Delta t' = 20.42 \times (2.2 \times 10^{-6} \text{ s}) = 4.49 \times 10^{-5} \text{ s}.$$

Not only can time dilation be observed between two reference frames, but also the length of a path in the resting frame (rest length L) will be longer than length measured in the moving frame (L') along the direction of movement. This follows directly from time dilation. For the classical case, the length L traveled can be found by

$$L = v \cdot \Delta t. \quad (2.15a)$$

To find the length traveled in the moving frame, L' , the previous reasoning on time dilation has to be reversed. The “observer” now appears in the moving frame, but the proper length (L) is measured in the stationary frame which now moves (in the opposite direction) with respect to that observer. The stationary observer measures both the proper time and the proper length, such that the moving observer consequently has to experience a properly contracted time interval. Effectively, the indices between the two reference frames have switched, such that now Eq. (2.12b) changes, and is rewritten as

$$\Delta t = \gamma \Delta t'. \quad (2.12c)$$

The length traveled in the moving frame then becomes

$$L' = v \cdot \Delta t'. \quad (2.15b)$$

Solving the above equation for $\Delta t'$ and substituting in Eq. (2.12c) for the moving frame of reference:

$$L' = v \cdot \Delta t' = v \frac{\Delta t}{\gamma} = \frac{L}{\gamma}, \quad (2.15c)$$

$$L = \gamma \cdot L', \quad (2.15d)$$

where $\gamma > 1$, $L > L'$, as the distance traveled must be longer in the rest frame.



EXAMPLE 2-1(b)

Find the distance that the muon in Example 2.1(a) travels in the rest frame, L . Note that the muon experiences the mean life in its rest frame (which in this example is the moving frame). The indices on t and t' are reversed.

Solution

From Example 2.1(a)

$$\gamma = 20.42,$$

$$\Delta t' = 2.2 \times 10^{-6} \text{ s (mean life that the muon experiences).}$$

Combining Eqs. (2.15b) and (2.15d),

$$L' = v \cdot \Delta t',$$

$$L = \gamma \cdot L' = \gamma \cdot v \cdot \Delta t',$$

$$L = 20.42 \cdot \left(0.9988 \cdot 3 \times 10^8 \frac{\text{m}}{\text{s}} \right) \cdot 2.2 \times 10^{-6} \text{ s},$$

$$L = 1.35 \times 10^4 \text{ m}.$$

The distance we would “expect” the muon to travel with nonrelativistic effects would be 20.42 times less than what we actually observe.

Velocity transformation follows, where u is the rest frame velocity, u' is the velocity in the primed reference frame, and the primed reference frame is moving at velocity v with respect to the rest frame:

$$u = \frac{u' + v}{\left(1 + u' \frac{v}{c^2} \right)}. \quad (2.16)$$

Relativistic momentum can thus be found by

$$\vec{p} = \gamma m \vec{v}, \quad (2.17)$$

and acceleration (note that acceleration decreases with increasing velocity) by

$$a = \frac{F}{m} \left(1 - \frac{u^2}{c^2} \right)^{3/2}. \quad (2.18)$$

The equivalence of mass and energy is one of the most important consequences of Einstein's special theory of relativity. According to Einstein, the relationship between mass and energy is

$$E = mc^2, \quad (2.19)$$

where E is the total energy of a piece of matter whose mass is m and c is the velocity of light in vacuum. Note that mc^2 is independent of velocity and is frequently referred to as “rest